Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

B:1/1/1/

Pearson

1. Given that a, b and c are integers greater than 0 such that

- $c = b + 2$
- $a + b + c = 10$

Prove, by exhaustion, that the product of a, b and c is always even.

You may use the table below to illustrate your answer.

(3)

You may not need to use all rows of this table.

a	b	c
6	1	3
4	2	4
2	3	5

Given :

$c = b + 2$

$a + b + c = 10 \rightarrow a = 10 - b - c$

We can use the two equations to find all possible values of a, b, c

(1) let $b = 1$: (as given)

$c = 1 + 2 = 3$

$a = 10 - 1 - 3 = 6$

(2) let $b = 2$: (as given)

$c = 2 + 2 = 4$

Question 1 continued

$a = 10 - 2 - 4 = 4$

(3) let $b = 3$:

→ note : if you try with $b \geq 4$, a will be ≤ 0, which doesn't satisfy the conditions provided in the question.

$c = 3 + 2 = 5$

$a = 10 - 3 - 5 = 2$

So :

when $a = 6$
$b = 1$
$c = 3$, $abc = 18 = 2(6)$

$a = 4$
$b = 2$
$c = 4$, $abc = 32 = 2(16)$

$a = 2$
$b = 3$
$c = 5$, $abc = 30 = 2(15)$

$\therefore$ All values of abc are even.

(Total for Question 1 is 3 marks)

2. A curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = (2 - kx)^5$$

and k is a constant.

Given that when $\mathrm{f}(x)$ is divided by $(4x - 5)$ the remainder is $\frac{243}{32}$

(a) show that $k = \frac{2}{5}$

(2)

(b) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{2}{5}x\right)^5$$

giving each term in simplest form.

(3)

Using the solution to part (b) and making your method clear,

(c) find the gradient of C at the point where $x = 0$

(2)

USE : REMAINDER THEOREM :

if $\frac{f(x)}{(ax-b)}$ = quotient + remainder :

then $f\left(\frac{b}{a}\right)$ = remainder

so : if $\frac{f(x)}{4x-5} = \frac{243}{32}$

then : $f\left(\frac{5}{4}\right) = \frac{243}{32}$

so : $\left(2 - \frac{5}{4}k\right)^5 = \frac{243}{32}$

$2 - \frac{5}{4}k = \sqrt[5]{\frac{243}{32}}$

$2 - \frac{5}{4}k = \frac{3}{2}$

Question 2 continued

$$-\frac{4}{5} \times \left(-\frac{5}{4}k\right) = \left(-\frac{1}{2}\right) \times -\frac{4}{5}$$

$$\therefore k = \frac{2}{5}$$

(b) Binomial expansion formula :

$$\boxed{(a+bx)^n = {}^nC_0 a^n (bx)^0 + {}^nC_1 a^{n-1}(bx)^1 + {}^nC_2 a^{n-2}(bx)^2 + \dots}$$

$${}^nC_r = \frac{n!}{(n-r)!\,r!}$$

n : power of binomial
r : (position of term) − 1

$\left(2 - \frac{2}{5}x\right)^5$ expanded becomes :

$${}^5C_0 (2)^5 \left(-\frac{2}{5}x\right)^0 + {}^5C_1 (2)^{5-1} \left(-\frac{2}{5}x\right)^1 + {}^5C_2 (2)^{5-2} \left(-\frac{2}{5}x\right)^2 + \dots$$

NOTE : • FIRST AND LAST ${}^nC_r$ ARE ALWAYS = 1
• WHEN $r = n$, then ${}^nC_r = n$

$$= (1)(32)(1) + 5(16)\left(-\frac{2}{5}\right)x + \frac{5!}{(5-2)!\,2!}(8)\left(\frac{4}{25}\right)x^2$$

$$= 32 - 32x + \frac{64}{5}x^2$$

(c) To find gradient, find the derivative of f(x).

$f(x) = \left(2 - \left(\frac{2}{5}\right)x\right)^2$ cannot be differentiated in this form, so we must use the expanded form.

Question 2 continued

so $f(x) = 32 - 32x + \frac{64}{5}x^2 + \ldots$

DIFFERENTIATION

$$\boxed{\begin{aligned} f(x) &= ax^n \\ f'(x) &= nax^{n-1} \end{aligned}}$$

$f'(x) = -32 + \frac{128}{5}x + \ldots$

$f'(0) = -32 + \frac{128}{5}(0) = -32$

Question 2 continued

(Total for Question 2 is 7 marks)

3. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_n = \cos^2\left(\frac{n\pi}{3}\right)$$

Find the exact values of

(a) (i) a_1

(ii) a_2

(iii) a_3

(3)

(b) Hence find the exact value of

$$\sum_{n=1}^{50}\left\{n + \cos^2\left(\frac{n\pi}{3}\right)\right\}$$

end

sum

terms

starting value

You must make your method clear.

(4)

(a) $a_n = \cos^2\left(\frac{n\pi}{3}\right)$

(i) $a_1 = \cos^2\left(\frac{1\pi}{3}\right) = \frac{1}{4}$

(ii) $a_2 = \cos^2\left(\frac{2\pi}{3}\right) = \frac{1}{4}$

(iii) $a_3 = \cos^2\left(\frac{3\pi}{3}\right) = 1$

(b) $\sum_{n=1}^{50}\left\{n + \cos^2\left(\frac{n\pi}{3}\right)\right\}$ means start the sequence at $n=1$ and sum all the terms up to $n=50$

↳ This sequence can be split into:

$$\sum_{n=1}^{50}\{n\} + \sum_{n=1}^{50}\left\{\cos^2\left(\frac{n\pi}{3}\right)\right\}$$

where $\sum_{n=1}^{50} n = \frac{n}{2}(a_1 + a_n) = \frac{50}{2}(1+50) = 1275$

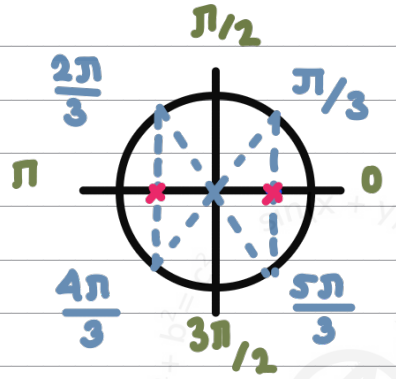
Question 3 continued

To find $\sum_{n=1}^{50}\left\{\cos^2\left(\frac{n\pi}{3}\right)\right\}$:

If you solve for a_4, you will find that :

$$a_4 = \cos^2\left(\frac{4\pi}{3}\right) = \frac{1}{4}$$

ON UNIT CIRCLE :



$\cos\left(\frac{2\pi}{3}\right), \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$

$\cos\left(\frac{\pi}{3}\right), \cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$

$\left(\pm\frac{1}{2}\right)^2 = \frac{1}{4}$

So, we know :

$a_1 = 1/4$
$a_2 = 1/4$
$a_3 = 1$
$a_4 = 1/4$

same value

Therefore the sequence repeats itself every 3 terms.

$$\underbrace{\frac{1}{4} + \frac{1}{4} + 1}_{} + \underbrace{\frac{1}{4} + \frac{1}{4} + 1}_{} + \dots$$

So : $\frac{50}{3 \text{ terms}} = 16 + 2$

repeats

remainder; add two more terms.

$$\sum_{n=1}^{50}\left\{\cos^2\left(\frac{n\pi}{3}\right)\right\} = 16\left(\frac{1}{4} + \frac{1}{4} + 1\right) + \frac{1}{4} + 1$$

$$= \frac{49}{2}$$

so : $\sum_{n=1}^{50}\left\{n + \cos^2\left(\frac{n\pi}{3}\right)\right\} = 1275 + \frac{49}{2} = \frac{2599}{2}$

(Total for Question 3 is 7 marks)

4. The weight of a baby mammal is monitored over a 16-month period.

The weight of the mammal, w kg, is given by

$$w = \log_a(t + 5) - \log_a 4 \qquad 2 \leqslant t \leqslant 18$$

where t is the age of the mammal in months and a is a constant.

Given that the weight of the mammal was 10 kg when $t = 3$

(a) show that $a = 1.072$ correct to 3 decimal places.

(3)

Using $a = 1.072$

(b) find an equation for t in terms of w

(3)

(c) find the value of t when $w = 15$, giving your answer to 3 significant figures.

(2)

(a) 1- Substitute $t = 3$, $w = 10$ into equation :

$$w = \log_a(t+5) - \log_a 4$$

$$10 = \log_a(3+5) - \log_a 4$$

2- Solve for a :

Recall rules of logarithms ;

$$\log_a(b) + \log_a(c) = \log_a(b \times c)$$

$$\log_a(b) - \log_a(c) = \log_a\left(\frac{b}{c}\right)$$

$$10 = \log_a 8 - \log_a 4$$

$$10 = \log_a\left(\frac{8}{4}\right)$$

RECALL !

$$\log_a x = b$$

$$a^b = x$$

$$a^{10} = 2$$

$$a = 2^{1/10} = 1.07177\ldots$$

$$= 1.072 \text{ (3 d.p.)}$$

Question 4 continued

(b) $w = \log_{1.072}(t+5) - \log_{1.072}(4)$

• make t subject of the formula :

$w = \log_{1.072}(t+5) - \log_{1.072}(4)$

$w = \log_{1.072}\left(\frac{t+5}{4}\right)$

$1.072^{w} = \frac{t+5}{4}$

• multiply both sides of eq. by 4 :

$4 \times \left(1.072^{w}\right) = \left(\frac{t+5}{4}\right) \times 4$

$4(1.072^{w}) = t+5$

• Subtract 5 from both sides :

$4(1.072^{w}) = t+5$

$(+5) \qquad (+5)$

$4(1.072^{w}) - 5 = t$

$t = 4(1.072^{w}) - 5$ → final equation ☺

(c) Substitute $w = 15$:

$t = 4(1.072^{15}) - 5 = 6.35$ (3 sf)

(Total for Question 4 is 8 marks)

5. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$(3\cos\theta - \tan\theta)\cos\theta = 2$$

can be written as

quadratic, only sinθ.

$$3\sin^2\theta + \sin\theta - 1 = 0$$

(3)

(b) Hence solve for $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$

$$(3\cos 2x - \tan 2x)\cos 2x = 2$$

(5)

(a) $(3\cos\theta - \tan\theta)\cos\theta = 2$

$3\cos^2\theta - \tan\theta\cos\theta = 2 \rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta}$

$3\cos^2\theta - \frac{\sin\theta}{\cancel{\cos\theta}}\cancel{\cos\theta} = 2$

$3\cos^2\theta - \sin\theta = 2$

Use identity: $\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$3(1 - \sin^2\theta) - \sin\theta = 2$

$3 - 3\sin^2\theta - \sin\theta = 2$

$3\sin^2\theta + \sin\theta - 1 = 0$

(b) As shown in (a):

can be written as:

$(3\cos 2x - \tan 2x)\cos 2x = 2$

$3\sin^2 2x + \sin 2x - 1 = 0$

FACTORISE:

Question 5 continued

$$3\sin^2 2x + 1\sin 2x - 1 = 0$$

$$\sin 2x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4(3)(1)}}{2(3)}$$

FOR: $-\pi \leq 2x \leq \pi$

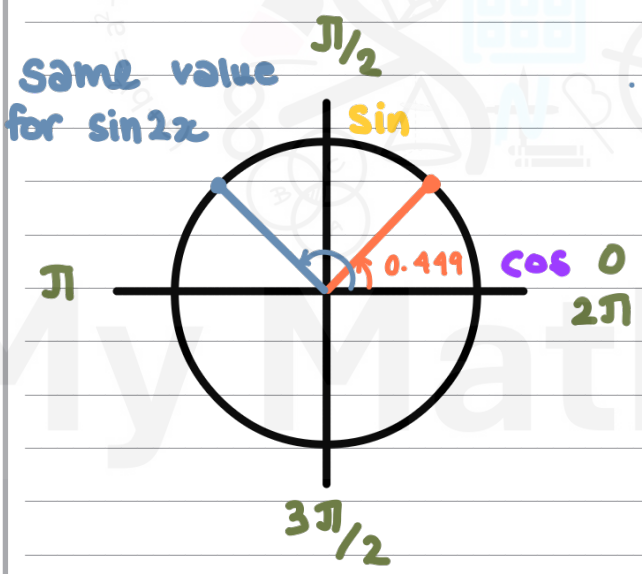
$$\sin 2x = \frac{-1 + \sqrt{13}}{6} \quad \text{or} \quad \sin 2x = \frac{-1 - \sqrt{13}}{6}$$

$$= 0.4343 \qquad\qquad -0.7676$$

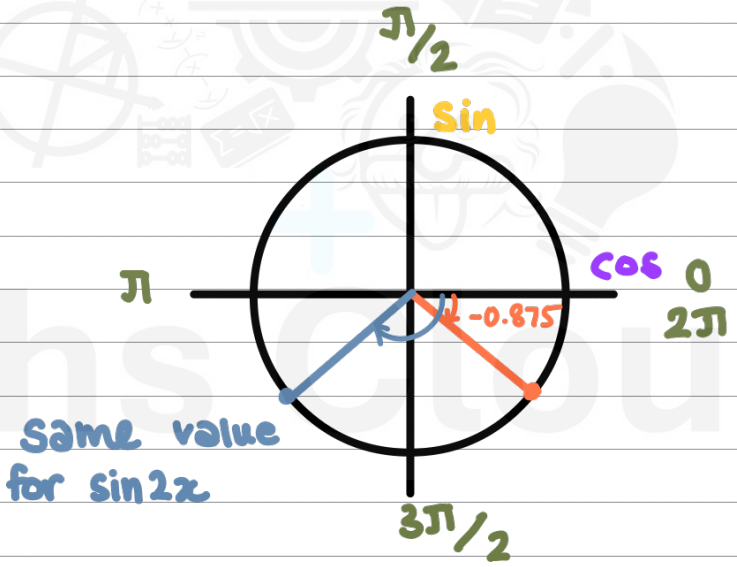
$$2x = \sin^{-1}\left(\frac{-1 + \sqrt{13}}{6}\right) \quad \text{or} \quad 2x = \sin^{-1}\left(\frac{-1 + \sqrt{13}}{6}\right)$$

$$2x = 0.449 \qquad\qquad 2x = -0.875$$

ON UNIT CIRCLE:



ON UNIT CIRCLE:



$$2x = \pi - 0.499 \qquad\qquad 2x = -\pi - (-0.875) =$$

$$= 2.64 \qquad\qquad = -2.27$$

So: $\frac{2x}{2} = \frac{-2.27}{2}, \frac{-0.875}{2}, \frac{0.449}{2}, \frac{2.64}{2}$

$$x = -1.13, -0.438, 0.225, 1.35$$

Question 5 continued

Question 5 continued

(Total for Question 5 is 8 marks)

6. The curve C_1 has equation $y = \mathrm{f}(x)$.

A table of values of x and y for $y = \mathrm{f}(x)$ is shown below, with the y values rounded to 4 decimal places where appropriate.

Interval btwn two x-coords = h of trapezium

x	0	0.5	1	1.5	2
y	3	2.6833	2.4	2.1466	1.92

y_1 y_6

multiplied by 2 as all are shared by two trapezia

(a) Use the trapezium rule with all the values of y in the table to find an approximation for

$$\int_0^2 \mathrm{f}(x)\,\mathrm{d}x$$

giving your answer to 3 decimal places.

(3)

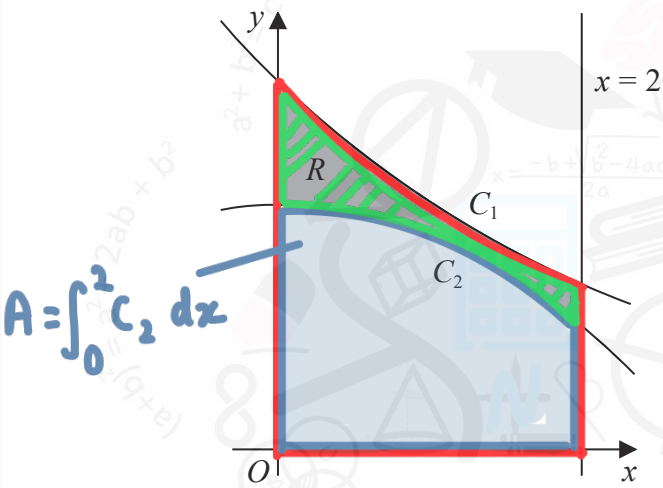


Figure 1

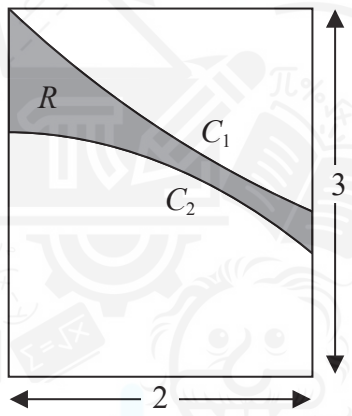


Figure 2

The region R, shown shaded in Figure 1, is bounded by

- the curve C_1
- the curve C_2 with equation $y = 2 - \frac{1}{4}x^2$
- the line with equation $x = 2$
- the y-axis

The region R forms part of the design for a logo shown in Figure 2.

The design consists of the shaded region R inside a rectangle of width 2 and height 3

Using calculus and the answer to part (a),

(b) calculate an estimate for the percentage of the logo which is shaded.

(4)

Question 6 continued

(a) (Formula is given in formula booklet.)

$$A = \frac{1}{2}h\left[y_1 + 2(y_2 + y_3 + y_4 + y_5) + y_6\right]$$

$h = 0.5 - 0 = 0.5$

$$A = \frac{1}{2}(0.5)\left[3 + 2(2.6833 + 2.4 + 2.1466) + 1.92\right]$$

$$= \frac{1}{4}(19.3798) = 4.84495$$

$$= 4.845 \text{ (3 d.p.)}$$

(b) A_{shaded} is A_R

$A_R = A_{under\ C_2} - A_{under\ C_1}$

$$A_R = \underbrace{\int_0^2 C_1\,dx}_{\text{estimated in part (a)}} - \int_0^2 C_2\,dx$$

$$A_R = 4.845 - \int_0^2 2 - \frac{1}{4}x^2\,dx$$

REMEMBER!

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1}}$$

$$A_R = 4.845 - \left[2x - \frac{x^3}{12}\right]_0^2$$

$$= 4.845 - 2(2) - \frac{(2)^3}{12}$$

Question 6 continued

$A_R = 4.845 - \frac{10}{3}$

% of logo shaded $= \frac{A_{shaded}}{A_{total}} \times 100$

$= \frac{4.845 - 10/3}{2 \times 3} \times 100$

$= 25.2\ \%$

Question 6 continued

(Total for Question 6 is 7 marks)

7. The curve C has equation

$$y = \frac{12x^3(x-7) + 14x(13x-15)}{21\sqrt{x}} \qquad x > 0$$

(a) Write the equation of C in the form

$$y = ax^{\frac{7}{2}} + bx^{\frac{5}{2}} + cx^{\frac{3}{2}} + dx^{\frac{1}{2}}$$

where a, b, c and d are fully simplified constants.

(3)

The curve C has three turning points.

Using calculus,

(b) show that the x coordinates of the three turning points satisfy the equation

$$2x^3 - 10x^2 + 13x - 5 = 0$$

means $f(1) = 0$

(3)

Given that the x coordinate of one of the turning points is 1

(c) find, using algebra, the exact x coordinates of the other two turning points.

(*Solutions based entirely on calculator technology are not acceptable.*)

(3)

(a) $y = \dfrac{12x^3(x-7) + 14x(13x-15)}{21\sqrt{x}}$

$y = \dfrac{12x^4 - 84x^3 + 182x^2 - 210x}{21x^{1/2}}$

$y = \dfrac{12x^4}{21x^{1/2}} - \dfrac{84x^3}{21x^{1/2}} + \dfrac{182x^2}{21x^{1/2}} - \dfrac{210x}{21x^{1/2}}$

simplify : $\dfrac{a^b}{a^c} = a^{b-c}$

$y = \dfrac{4}{7}x^{7/2} - 4x^{5/2} + \dfrac{26}{3}x^{3/2} - 10x^{1/2}$

$a = 4/7$ $\quad c = 26/3$

$b = -4$ $\quad d = -10$

Question 7 continued

(b) At a turning point, $dy/dx = 0$

$$y = \frac{4}{7}x^{7/2} - 4x^{5/2} + \frac{26}{3}x^{3/2} - 10x^{1/2}$$

DIFFERENTIATION

$$\boxed{\begin{aligned} y &= ax^n \\ \frac{dy}{dx} &= nax^{n-1} \end{aligned}}$$

$$\frac{dy}{dx} = \frac{\cancel{7}}{\cancel{2}}\left(\frac{\cancel{4}^{2}}{\cancel{7}}\right)x^{7/2-1} - \frac{\cancel{5}}{\cancel{2}}(\cancel{4}^{2})x^{5/2-1} + \frac{\cancel{3}}{\cancel{2}}\left(\frac{\cancel{26}^{13}}{\cancel{3}}\right)x^{3/2-1} - \frac{1}{\cancel{2}}(\cancel{10}^{5})x^{1/2-1}$$

$$\frac{dy}{dx} = 2x^{5/2} - 10x^{3/2} + 13x^{1/2} - 5x^{-1/2} = 0$$

$$x^{1/2}\cdot\left(2x^{5/2} - 10x^{3/2} + 13x^{1/2} - 5x^{-1/2}\right) = (0)\cdot x^{1/2}$$

$$2x^{6/2} - 10x^{4/2} + 13x^{2/2} - 5x^{0/2} = 0$$

$$2x^3 - 10x^2 + 13x - 5 = 0$$

(c) If $f(1) = 0$, then $(x-1)$ is a factor of $f(x)$.
Since $f(x) = (x-1)$(another expression), use algebraic division to find it.

$$x-1 \;\big)\; 2x^3 - 10x^2 + 13x - 5$$

(1) Divide first term of dividend by first term of divisor: $\frac{2x^3}{x} = 2x^2$

$$\begin{array}{r|l} & 2x^2 \\ \hline x-3 & 2x^3 - 10x^2 + 13x - 5 \end{array}$$

$\div$

$\hookrightarrow$ $2x^2$ is the first term of our quotient.

Question 7 continued

(2) Multiply out and write under:

$$\begin{array}{r|l} & 2x^2 \\ x-1 & 2x^3-10x^2+13x-5 \\ & 2x^3-2x^2 \end{array}$$

(3) Subtract then bring down next term:

$$\begin{array}{r|l} & 2x^2 \\ x-1 & 2x^3-10x^2+13x-5 \\ & -(2x^3-2x^2) \\ & -8x^2+13x \end{array}$$

(4) Divide first term of expression by first term of divisor: $\frac{-8x^2}{x} = -8x$

$$\begin{array}{r|l} & 2x^2-8x \\ x-1 & 2x^3-10x^2+13x-5 \\ & -(2x^3-2x^2) \\ & -8x^2+13x \end{array}$$

(5) Multiply out and write under:

$$\begin{array}{r|l} & 2x^2-8x \\ x-1 & 2x^3-10x^2+13x-5 \\ & -(2x^3-2x^2) \\ & -8x^2+13x \\ & -8x^2+8x \end{array}$$

(6) Subtract then bring down next term:

$$\begin{array}{r|l} & 2x^2-8x \\ x-1 & 2x^3-10x^2+13x-5 \\ & -(2x^3-2x^2) \\ & -8x^2+13x \\ & -(-8x^2+8x) \\ & 5x-5 \end{array}$$

Question 7 continued

(7) Divide first term of expression by first term of divisor : $\frac{5x}{x} = 5$

$$
\begin{array}{r|l}
 & 2x^2 - 8x + 5 \\
\hline
x-1 & 2x^3 - 10x^2 + 13x - 5 \\
 & -(2x^3 - 2x^2) \\
\hline
 & -8x^2 + 13x \\
 & -(-8x^2 + 8x) \\
\hline
 & 5x - 5
\end{array}
$$

(8) Multiply out and write under :

$$
\begin{array}{r|l}
 & 2x^2 - 8x + 5 \\
\hline
x-1 & 2x^3 - 10x^2 + 13x - 5 \\
 & -(2x^3 - 2x^2) \\
\hline
 & -8x^2 + 13x \\
 & -(-8x^2 + 8x) \\
\hline
 & 5x - 5 \\
 & 5x + 5
\end{array}
$$

(9) Subtract :

$$
\begin{array}{r|l}
 & 2x^2 - 8x + 5 \\
\hline
x-1 & 2x^3 - 10x^2 + 13x - 5 \\
 & -(2x^3 - 2x^2) \\
\hline
 & -8x^2 + 13x \\
 & -(-8x^2 + 8x) \\
\hline
 & 5x - 5 \\
 & -(5x + 5) \\
\hline
 & 0
\end{array}
$$

So $f(x) = (x-1)(2x^2 - 8x + 5)$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(5)}}{2(2)}$$

$$= \frac{4 \pm \sqrt{6}}{2}$$

(Total for Question 7 is 9 marks)

8. A geometric sequence has first term a and common ratio r

Given that $S_\infty = 3a$ sum to infinity

(a) show that $r = \frac{2}{3}$

(2)

Given also that

$$u_2 - u_4 = 16$$

where u_k is the kth term of this sequence,

(b) find the value of S_{10} giving your answer to one decimal place.

(5)

(a) Use sum to infinity formula :

$\Rightarrow S_\infty$ converges ONLY if : $-1 < r < 1$

where $S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{\cancel{a}}{1-r} = 3\cancel{a}$

$\frac{1}{1-r} = \frac{3}{1}$

$1 - r = \frac{1}{3}$

$\cancel{+}r = \cancel{+}\frac{2}{3} \rightarrow r = \frac{2}{3}$

(b) In a geometric sequence, any term (u) can be described by :

$u_k = u_1 r^{k-1}$

we know : $u_1 = a$

$r = \frac{2}{3}$

$u_2 - u_4 = 16$

Question 8 continued

$$a\left(\frac{2}{3}\right)^{2-1} - a\left(\frac{2}{3}\right)^{4-1} = 16$$

$$\frac{2}{3}a - \frac{8}{27}a = 16$$

$$\frac{10}{27}a = 16 \longrightarrow a = 43.2$$

Use partial sum formula :

$$S_k = \frac{u_1(1-r^k)}{(1-r)}$$

$$S_{10} = \frac{43.2(1-(2/3)^{10})}{(1-2/3)} = 127.4 \text{ (1 d.p.)}$$

(Total for Question 8 is 7 marks)

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

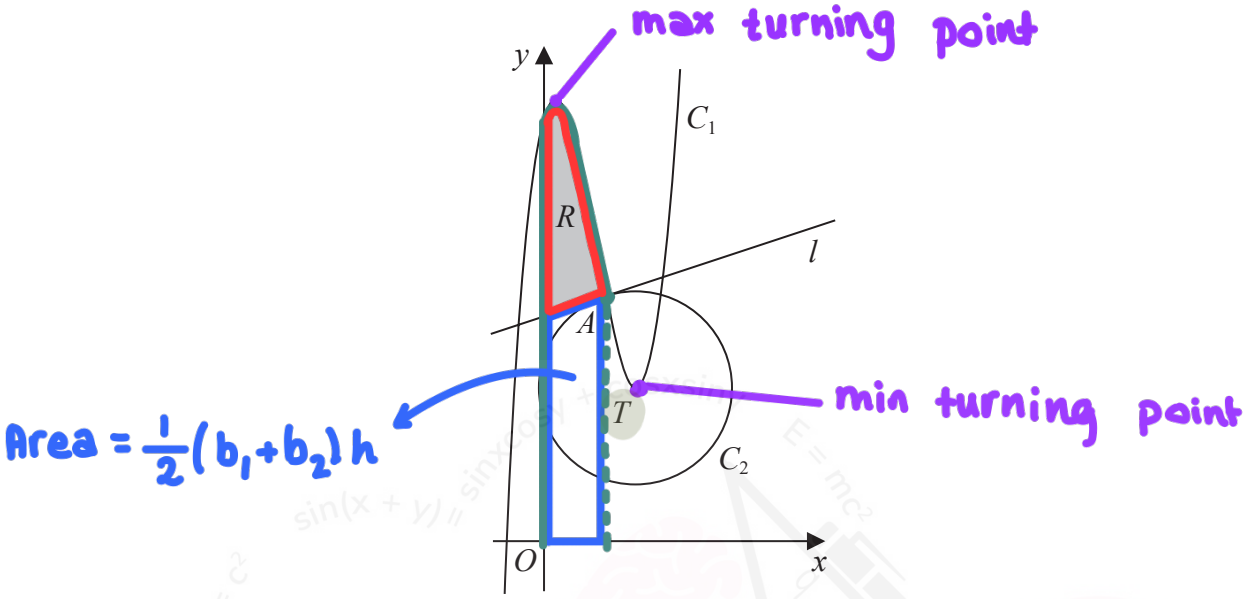


Figure 3

Figure 3 shows

- the curve C_1 with equation $y = x^3 - 5x^2 + 3x + 14$
- the circle C_2 with centre T $\quad dy/dx = 0$

The point T is the minimum turning point of C_1

Using Figure 3 and calculus,

(a) find the coordinates of T

(3)

The curve C_1 intersects the circle C_2 at the point A with x coordinate 2

(b) Find an equation of the circle C_2

(3)

The line l shown in Figure 3, is the tangent to circle C_2 at A

(c) Show that an equation of l is

$$y = \frac{1}{3}x + \frac{22}{3}$$

(3)

The region R, shown shaded in Figure 3, is bounded by C_1, l and the y-axis.

(d) Find the exact area of R.

(3)

Question 9 continued

(a) $y = x^3 - 5x^2 + 3x + 14$

DIFFERENTIATION

$y = ax^n$

$\frac{dy}{dx} = nax^{n-1}$

$\frac{dy}{dx} = 3x^{3-1} - (2)5x^{2-1} + (1)3x^{1-1}$

$= 3x^2 - 10x + 3$

At a turning point, $\frac{dy}{dx} = 0$

$3x^2 - 10x + 3 = 0$ (FACTORISE)

$(3x - 1)(x - 3) = 0$

$x = \frac{1}{3}$, $x = 3$ are both turning points.

By using the graph, you can see that $x = \frac{1}{3}$ is a maximum, while $x = 3$ is a minimum.

y-coordinate at $x = 3$:-

$y = 3^3 - 5(3)^2 + 3(3) + 14$

$= 5$

So coords T $= (3, 5)$

(b) Eq. of circle :

$(x - x_{centre})^2 + (y - y_{centre})^2 = r^2$

We know :

$x = 2$

$x = 3$

$y = 5$

We have to find :

$y = ?$

$r^2 = ?$

Question 9 continued

① Find y by substituting $x = 2$ into C_1

$$y = x^5 - 5x^2 + 3x + 14$$

$$2^5 - 5(2)^2 + 3(2) + 14 = 8$$

② Find r^2 by using distance formula :

$$(r)^2 = \left(\sqrt{(x - x_c) + (y - y_c)}\right)^2$$

$$r^2 = (2-3)^2 + (8-5)^2 = 10$$

so eq. of circle :

$$(x-3)^2 + (y-5)^2 = 10$$

(c) The tangent (ℓ) of the circle at A is perpendicular to the radius. so :

$$m_{tangent} = -\frac{1}{m_{radius}}$$

$$m_{radius} = \frac{8-5}{2-3} = -3$$

$$m_{tangent} = -\frac{1}{(-3)} = \frac{1}{3}$$

$$y = mx + c$$

substitute coordinates of A : $(2, 8)$

$$8 = \frac{1}{3}(2) + c \rightarrow c = \frac{22}{3}$$

Question 9 continued

So $y = \frac{1}{3}x + \frac{22}{3}$

(d) $A_R = \int_0^2 x^3 - 5x^2 + 3x + 14 \, dx - A_{trapezium}$

REMEMBER!

$$\boxed{\int x^n \, dx = \frac{x^{n+1}}{n+1}}$$

$$A_R = \left[\frac{x^4}{4} - \frac{5}{3}x^3 + \frac{3}{2}x^2 + 14x \right]_0^2 - \frac{1}{2}\left(\frac{22}{3} + 8\right)(2)$$

$$A_R = \frac{2^4}{4} - \frac{5}{3}2^3 + \frac{3}{2}2^2 + 14(2) - \frac{1}{2}\left(\frac{22}{3} + 8\right)(2)$$

$$A_R = \frac{74}{3} - \frac{46}{3} = \frac{28}{3}$$

(Total for Question 9 is 12 marks)

10. Given $a = \log_2 3$

(i) write, in simplest form, in terms of a,

(a) $\log_2 9$

(b) $\log_2\left(\frac{\sqrt{3}}{16}\right)$

(3)

(ii) Solve

$$3^x \times 2^{x+4} = 6$$

giving your answer, in simplest form, in terms of a.

(4)

(i)(a) $\log_2 9 = \log_2 3^2$

$= 2\log_2 3$

$= 2a$

$\boxed{\log x^a = a\log x}$

(b) $\log_2\left(\frac{\sqrt{3}}{16}\right)$

Recall rules of logarithms:

$\log_a(b \times c) = \log_a(b) + \log_a(c)$

$\log_a\left(\frac{b}{c}\right) = \log_a(b) - \log_a(c)$

$= \log_2\sqrt{3} - \log_2 16$

$= \log_2 3^{1/2} - \log_2 2^4$

$\left(\log x^a = a\log x\right)$

$= \frac{1}{2}\log_2 3 - 4\log_2 2$

$\left(\log_a a = 1\right)$

$= \frac{1}{2}a - 4(1)$

$= \frac{1}{2}a - 4$

Question 10 continued

(ii) $3^x \times 2^{x+4} = 6$

METHOD 1:

$\log_2(3^x \times 2^{x+4}) = \log_2 6$

$\log_2 3^x + \log_2 2^{x+4} = \log_2(2 \times 3)$

$x\log_2 3 + (x+4)\log_2 2 = \log_2 2 + \log_2 3$

$x\log_2 3 + x(1) + 4(1) = (1) + \log_2 3$

$x(\log_2 3 + 1) = \log_2 3 - 3$

$x(a+1) = a-3$

$x = \dfrac{a-3}{a+1}$

METHOD 2:

$\log_2(3^x \times 2^x \times 2^4) = \log_2(3 \times 2)$

$\log_2 3^x + \log_2 2^x + \log_2 2^4 = \log_2 3 + \log_2 2$

$x\log_2 3 + x\log_2 2 + 4\log_2 2 = \log_2 3 + \log_2 2$

$x\log_2 3 + x(1) + 4(1) = \log_2(3) + 1$

$x(\log_2 3 + 1) = \log_2 3 - 3$

$x(a+1) = a-3$

$x = \dfrac{a-3}{a+1}$

Question 10 continued

METHOD 3:

$$\left[\frac{a^x}{a^b} = a^{x-b}\right]$$

$$\frac{3^x \times 2^{x+4}}{3^1 \times 2^1} = \frac{3^1 \times 2^1}{3^1 \times 2^1}$$

$$3^{x-1} \times 2^{x+3} = 1$$

$$\log_2(3^{x-1} \times 2^{x+3}) = \log_2 1 \qquad (\log_x 1 = 0)$$

$$\log_2 3^{x-1} + \log_2 2^{x+3} = 0$$

$$(x-1)\log_2 3 + (x+3)\log_2 2 = 0$$

$$(x-1)a + x + 3 = 0$$

$$xa - a + x + 3 = 0$$

$$x(a+1) = a - 3$$

$$x = \frac{a-3}{a+1}$$

(Total for Question 10 is 7 marks)

TOTAL FOR PAPER IS 75 MARKS